Phil 2310
Fall 2010

Assignment 6: This homework is due by the beginning of class on Fri, Oct 15th.

Part I: Practicing Taut Con

Show that each of the following arguments is valid by constructing a proof in $\mathcal{F}$. You should write out each proof on a piece of paper and hand it in to me in class. You could also write your proof in Fitch and then simply print it out or email them to me. If you do that, you must click 'show step numbers' and also 'verify proof' before you print it.

You may use any rules of $\mathcal{F}_T$ plus you can use Taut Con for any step that I consider to be sufficiently obvious. This will be a judgment call so err on the side of caution (and use other rules). If it is something we explicitly mentioned in class, that is okay. Below are other steps that are okay uses of Taut Con. Some of these will be helpful for the problems and the problems are written to get you to use some of these.

Modus Tollens
$P \rightarrow Q, \neg Q \vdash \neg P$

Conditionals
$\neg P \vee Q \Leftrightarrow P \rightarrow Q$
$P \wedge \neg Q \Leftrightarrow \neg(P \rightarrow Q)$

Disjunctive Syllogism
$P \vee Q, \neg P \vdash Q$

DeMorgan's Laws
$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$
$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$

Biconditionals
$P \leftrightarrow Q, \neg P \vdash \neg Q$
$P \leftrightarrow Q \Leftrightarrow \neg P \leftrightarrow \neg Q$
$\neg(P \leftrightarrow Q) \Leftrightarrow \neg P \leftrightarrow Q$

1. $P \vee Q \vdash (\neg Q \rightarrow \neg P) \rightarrow Q$
2. $P \rightarrow Q, \neg P \rightarrow R \vdash Q \vee R$
3. $\neg(P \wedge Q), \neg(\neg P \wedge Q) \vdash \neg Q$
4. $(P \rightarrow Q) \rightarrow P, P \rightarrow R \vdash R$
5. $(P \rightarrow Q) \rightarrow Q \vdash (Q \rightarrow P) \rightarrow P$
6. $(P \rightarrow Q) \vee (R \rightarrow S) \vdash (P \rightarrow S) \vee (R \rightarrow Q)$
7. $P \leftrightarrow (Q \leftrightarrow R) \vdash (\neg Q \wedge \neg P) \rightarrow R$

Part II. Write a sentence using only ¬ and ∨ as connectives that is equivalent to the following sentence: $(P \rightarrow Q) \leftrightarrow (\neg S \wedge R)$

[Hint – think about some of the above equivalences]

Part III: Propositional Logic Metatheory

1. Consider the made-up rule ∨IA:
   (for ∨Intro in the antecedent)

| k. $X \to Z$
|———
| l. $(X \lor Y) \to Z$ ∨IA k.

Would this be an acceptable short-cut rule to add to our proof system? Why or why not? Would the Soundness and Completeness Theorems still be true of this new system?

2. Consider the made-up rule ∨CA:
   (for ∨ chain argument)

| k. $X \lor Y$
| m. $Z \lor \neg Y$
| ———
| n. $X \lor Z$ ∨CA k,m

Would this be an acceptable short-cut rule to add to our proof system? Why or why not? Would the Soundness and Completeness Theorems still be true of this new system?

Part IV.
Read Chapter 9 in our book.
Do problems 9.16 and 9.17 (lots of translations)